

TABLE IV

$p$ -level ( $N = 2, l = 1$ )		
$E$ atomic units	$r_0$ atomic units	from section
-0.1250	$\infty$	3a
-0.1193	10.235	3b
-0.1097	8.540	3b
-0.0895	7.089	3b
-0.0800	6.701	3b
-0.0740	6.497	3b
-0.0566	6.000	3a
-0.0408	5.696	3b
-0.0312	5.528	3a
-0.0200	5.355	3a
0	5.086	3c
0.0312	4.770	3d
0.0556	4.554	3d
0.1250	4.110	3d
0.5000	2.698	3d
0.8261	2.528	3d
2.524	1.68	3f
3.116	1.55	3f
3.943	1.41	3f
$\infty$	0	3e

the points  $r = 0$  and  $r = \infty$ . For several and 1 zero points have been calculated and publication § 3a) and represented in the figure splitting up of the second level into 2s. It is obvious that by using only integer in the curves is left between  $n = l + 1$  when  $r_0 = \infty$  and a comparatively small large radii  $r_0$  this gap could be filled by the gives only approximative values. gap in the curve it is necessary to find a confluent hypergeometric function with real interpolated from tables 6) 7) with the solution procedures (v. tables II-IV and

case of  $n \rightarrow \infty$  has been studied by Welker<sup>2)</sup>, especially for the 1s level. The function (6) for  $n \rightarrow \infty$  is proportional

$$J_{2l+1}(2\sqrt{\varphi n}) = J_{2l+1}(2\sqrt{2r}). \quad (24)$$

The first and second node of  $J_1$  give the values for the 1s and 2s level. The first zero point of  $J_3$  gives the intersection of the 2p level with the  $r_0$ -axis. (v. tables II-IV and figures).

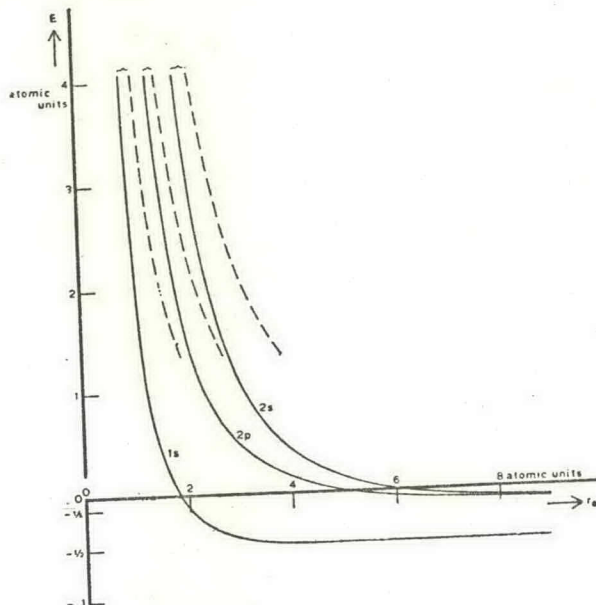


Fig. 3. The  $(E, r_0)$ -curves for the 1s, 2s and 2p-levels. Asymptotes are the dotted lines and  $r_0 = 0, E = -0.5$  (for the 1s curve) and  $E = -0.125$  (for the 2s and 2p curves).

Sommerfeld and Welker stressed the importance of a more general investigation of the behaviour of confluent hypergeometric functions  $F$  in the neighbourhood of  $n = \infty$  or  $E = 0$ . For that purpose function  $F$  of equation (6) must be expanded as a power-series in  $n^{-1}$ . By the definition of  $F$  (7) and with (6) and (2) the wave function can be written:

$$F(l+1-n, 2l+2, 2rn^{-1}) = \sum_{k=0}^{\infty} \frac{(-1)^k (2r)^k (2l+1)!}{k! (2l+k+1)!} \prod_{v=1}^k \{1-(l+v)n^{-1}\}. \quad (25)$$

The product can be written as the sum  $\sum_{v=0}^k (-1)^v a_v^{k,l} n^{-v}$  where  $a_v^{k,l}$  is the sum of the  $\binom{k}{v}$  products of  $v$  different numbers of the series  $l+1, l+2, \dots, l+k$  (without repetitions). The first three \*) are

$$a_0^{k,l} = 1, \quad (26)$$

$$a_1^{k,l} = \frac{1}{2}k(k+2l+1), \quad (27)$$

$$a_2^{k,l} = k(k-1) \left\{ \frac{1}{2}l^2 + \frac{1}{2}(k+1)l + \frac{1}{24}(k+1)(3k+2) \right\}. \quad (28)$$

\*) The Newton relations<sup>2)</sup> that can eventually be used to calculate these coefficients are of course also valid here.